Radial Flow Gas Turbines

Radial outflow turbine

(a) Meridional section through turbine
(b) Blading arrangement and directions of rotation
Radial Flow Gas Turbines

Radial Inflow Turbines

These turbines provide greater amount of work per stage, ease of manufacture, and ruggedness. Efficiency is equal to the best of those of axial turbines.

Main disadvantage is requirement of advanced techniques for rotor cooling and shock resistance.

Commonly used in automobile turbochargers and helicopters, and expanders for cryogenic processes and liquefaction of gases.

1. Cantilever turbine
Radial Flow Gas Turbines

There is zero incidence flow due to the direction of the relative velocity which makes this an inward flow turbine.

Working principle is the reverse of a centrifugal compressor with different impeller shapes.

1. 90° IFR (Centripetal) Turbine
Radial Flow Gas Turbines

\[
\frac{1}{2} c_1^2 = h_1 + h_{t1} = h_{t2} + h_{t3} + h_{t4} + h_{t5} + h_{t6} + h_{t7} + h_{t8}
\]

\[h_{1} - h_{3} \] -> ideal enthalpy change

\[h_{0\text{rel}} = h_{0\text{rel}} - h_{0\text{rel}} = \left(\frac{1}{2} U_2^2 - \frac{1}{2} U_3^2\right) \rightarrow \text{ideal}
\]

\[
\Rightarrow h_4 - h_3 = \frac{1}{2} \left[U_2^2 - U_3^2\right] - \left(U_2^2 - U_3^2\right) \rightarrow \text{ideal}
\]

\[h_4 - h_3 = \frac{1}{2} \left(c_3^2 - c_4^1\right)
\]

\[\Delta W = h_{01} - h_{05} = \frac{1}{2} \left[U_2^2 - U_3^2\right] \rightarrow \text{specific}
\]

\[
= \frac{1}{2} \left[U_2^2 - U_3^2\right] - \left(U_3^2 - U_4^2\right) + \left(c_3^2 - c_4^1\right)
\]

- Acceleration of relative velocity is desired for higher specific work.
- \((\Delta W)\) brings a significant contribution to specific work.

Nominal design: \(w_2 = c_{02} \Rightarrow c_{02} = U_2\)

\(c_3 = c_{03} \Rightarrow c_{03} = 0\)

\[\Delta W = \frac{U_2^2}{2}\]
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Rotor of end 1F2 turbine rotates at 38,160 rpm.

With 23.76 cm, absolute flow angle at rotor inlet is 92° and rotor mean exit diameter is one half of the rotor diameter, and rotor exit relative velocity is twice the rotor inlet relative velocity. Calculate the specific work and ratio of contributions of $\Delta W$, $\Delta w$, and $\Delta c$.

$w_2 = \pi DN / 60 = 494.5 \text{ m/s}$

Referring to the inlet velocity triangle:

$w_2 = w_2 \cos \theta_2 = 154.19 \text{ m/s}$

$c_2 = \sqrt{w_2^2 + w_2^2} = 498.9 \text{ m/s}$

$c_j = \sqrt{w_j^2 - w_2^2} = \sqrt{(2 \times 154.19)^2 - (498.9)^2} = 239.6 \text{ m/s}$

$\Delta W = 168.863 \text{ m}^2 \text{ s}^2$/

$\Delta w = 71.305 \text{ m}^2$/

$\Delta c = 210.415 \text{ m}^2$/

$\frac{\Delta c}{\Delta w} = 29.5\%$

$R_{w} = \frac{\Delta w}{\Delta W} = 15.8\%$

$R_{c} = \frac{\Delta c}{\Delta W} = 46.69\%$

$\beta_2 = \frac{c_2}{w_2} = C_{r2}$
Radial Flow Gas Turbines

Spouting Velocity: A velocity that its kinetic energy is equal to the isentropic enthalpy drop from the inlet stagnation pressure to the exhaust pressure:

\[ \frac{1}{2} \alpha_0^2 = h_{01} - h_{02} = \gamma \text{To} \left[ \frac{1}{2} \rho_1 \left( \frac{P_1}{T_1} \right)^{\kappa} \right] \]

For ideal case at normal conditions:

\[ \Delta \psi = \frac{U_2}{U_1} = \frac{1}{2} \alpha_0^2 \]

Therefore:

\[ \frac{U_2}{U_0} = 0.9707 \]

This ratio is in the range of 0.68 < \( \frac{U_2}{U_0} < 0.71 \)

Nominal Design Point Efficiency:

\[ \eta_{ns} = \left[ 1 + \frac{1}{2} \left( \alpha_2^2 + w_2^2 \cos \alpha_2 + c_2^2 \frac{T_2}{T_1} \right) \Delta \psi \right]^{-1} \]

\[ c_2 = U_2 \csc \alpha_2 \]

\[ w_2 = U_2 \csc \beta_2 \]

\[ \Delta \psi = \frac{U_2}{U_1} \]

\[ U_2 = U_1 \left( \frac{T_2}{T_1} \right)^{\frac{\kappa}{2}} \]

\[ \eta_{ns} = \left[ 1 + \frac{1}{2} \left( \delta_2 \frac{T_2}{T_1} \csc \alpha_2 + \left( \frac{T_2}{T_{1IC}} \right)^{\frac{\kappa}{2}} \delta_2 \csc^2 \beta_2 + \cot \beta_2 \right) \right]^{-1} \]
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\[ q_{tt} = \frac{\Delta w}{2} \frac{\Delta \psi_0 - \frac{1}{2} c_3^2}{2 \Delta w} = \frac{1}{q_{tt}} - \frac{c_3^2}{2 \Delta w} \]

Finally:

\[ \frac{1}{q_{tt}} = \frac{1}{q_{tt}} - \frac{c_3^2}{2 \Delta w} + \frac{1}{2 \Delta w} \left( \frac{c_3 + \sqrt{c_3^2 + 8 c_2}}{2} \right)^2 \]

Example:

Below data is given for a radial turbine:

- Rotor inlet diameter: 92.5 mm
- Rotor inlet width: 9.14 mm
- Mean outlet diameter: 94.4 mm
- Outlet annulus height: 10.1 mm
- Rotor inlet angle: 0°
- " " outlet " ": 50°
- " " of rotor blades: 10
- Nozzle outlet diameter: 74.1 mm
- " " angle: 80°
- Blade number: 15

\[ T_{r1} = 400 K \]

\[ \frac{m \sqrt{T_{r1}/P_{r1}}}{1.44 \times 10^{-5} m^3/K} = 2410 \text{ rev/min} \]

\[ c_{r1}/P_{r1} = 4.59 \times 10^{-6} m^3 \]

a) Calculate the total-to-static and overall efficiencies.

b) The rotor loss curve coefficient.
Radial Flow Gas Turbines

\[ h_a - h_{33} = C_p \frac{\pi}{2} \left[ 1 - \left( \frac{P_2}{P_{t01}} \right)^{\frac{k-1}{2}} \right] = 53.97 \text{ kJ/kg} \]

\[ N = 2410 \sqrt{\frac{\pi}{2}} = 68.2 \text{ kW/m} \]

\[ \omega_2 = \frac{N D_2}{60} = 18.3 \text{ rad/s} \]

\[ \Delta \omega = \omega_2^2 = 73.48 \text{ rad/s} \]

\[ \frac{\Delta \omega}{\Delta \omega_0} = \frac{72.48}{47.99} = 76.14\% \]

\[ \Delta \omega_{act} = \frac{\pi \Omega}{m} = \left( \frac{\pi}{P_{t01}} \right) \left( \frac{\pi}{m \sqrt{\frac{T_01}{P_{t01}}} \right) \Delta \omega_0 \]

\[ = 4.59 \times 10^{-6} \times 2410 \times \pi \times 400 \]

\[ = 22.18 \text{ kJ/kg} \]

\[ \frac{\Delta \omega_{act}}{\Delta \omega} = 73.18\% \]
Mach Number Relations

\[ M_2 = \frac{C_2}{a_2} = \frac{U_2 \cos \alpha_2}{a_2} \]

\[ T_2 = T_01 - \frac{C_e^2}{2C_p} = T_01 - \frac{1}{2} \frac{U_2^2 \cos^2 \alpha_2}{C_P} \]

\[ \frac{T_2}{T_01} = 1 - \frac{1}{2} \left( k-1 \right) \left( \frac{U_2}{a_01} \right)^2 \cos^2 \alpha_2 \]

and \[ a_2 = a_{01} \left( \frac{T_2}{T_01} \right)^{1/2} \]

\[ M_2 = \frac{U_2/a_{01}}{\sin \alpha_2 \left[ 1 - \frac{1}{2} \left( k-1 \right) \left( \frac{U_2}{a_01} \right)^2 \cos^2 \alpha_2 \right]^{1/2}} \]

At rotor outlet:

\[ M_{3,rel} = \frac{\left( U_2/a_{01} \right) \left( r_3/r_2 \right)}{\sin \beta_3 \left[ 1 - \left( k-1 \right) \left( U_2/a_{01} \right)^2 \left[ 1 + \frac{1}{2} \left( \frac{r_1}{r_2} \cos \beta_3 \right)^2 \right] \right]} \]
Radial Flow Gas Turbines

Loss coefficients in 90° IFR Turbine

\[ \delta_N = \left( h_2 - h_{1s} \right) / \frac{1}{2} c_i^2 \]

\[ \phi_N = c_2 / c_{1s} \rightarrow \text{isentropic velocity coeff.} \]

\[ \gamma_N = \frac{(P_{out} - P_{in})}{(A_2 - A_1)} \rightarrow \text{stagnation loss coeff.} \]

\[ \gamma_N \approx S_N \left( 1 + \frac{1}{2} \frac{1}{M_2^2} \right) \]

\[ 0.9 < \phi_N < 0.97 \]
\[ 0.25 < \delta_N < 0.063 \]

For the rotor

\[ \delta_2 = \frac{1}{\phi_2} - 1 \]

0.70 < \phi_2 < 0.85

1.04 < \delta_2 < 0.88

\[ \phi_2 = \frac{w_2}{w_{3s}} \]

Optimum efficiency

\[ \lambda = \frac{c_o^2}{U^2} \rightarrow \text{incidence factor (and ups to slip factor)} \]

\[ \lambda = 1 - 0.65 \frac{\pi}{2} \approx 1 - \frac{2}{3} \]

\[ \theta_2 = \left( \frac{2}{3} \right) \frac{w_2}{c_{m2}} \]

Non-dimensional power ratio:

\[ S = \frac{\Delta W}{h_{01}} = 1 - \frac{T_{21}}{T_{01}} \]

\[ \tau = \frac{S}{1 - (p_{B01})^{1/2}} \]

Criterion for minimum blade number (Z)

\[ Z_{min} = 2\pi + \delta_2 \rightarrow \text{Jameson} \]

\[ Z = \frac{\pi}{30} (10 - \alpha_2) \tan d_2 \rightarrow \text{Blissman} \]
Radial Flow Gas Turbines

Example: An IFR turbine with 12 vanes is required to develop 230 kW from a supply of dry air available at stagnation temperature of 1050 K at 1 kg/s. If \( \theta_{ts} = 0.81 \), calculate:
(a) absolute and relative flow angles at rotor inlet
(b) overall pressure ratio \( \left( \frac{P_3}{P_{01}} \right) \) \( \theta_t = 1.333, \) \( \theta_t = 1150 \) J/kg

\[ a) \quad s = \frac{\Delta \omega}{\omega_{21}} = \frac{230}{115 \times 0.55} = 0.2 \]

\[ \cos^2 \alpha_2 = \frac{\gamma}{\gamma - 1} = \frac{1.4}{1.4 - 1} = 0.8333 \]

\[ \Rightarrow \alpha_2 = 33.22^\circ \]

\[ 2 \cos^2 \beta_2 - 1 = -\cos \beta_2 \Rightarrow \]

\[ \beta_2 = 33.56^\circ \]

\[ b) \quad \frac{P_3}{P_{01}} = \left( \frac{1 - s}{\theta_{ts}} \right)^{\frac{1}{\gamma - 1}} = \left( 1 - \frac{0.2}{0.81} \right)^{1.4} \]

\[ = 0.32165 \]
Radial Flow Gas Turbines

Example

For the given data of a radial gas turbine, determine:

a) the rotor diameter at rotational speed
b) the width at diameter ratio \( z_2/z_1 \) at rotor inlet

\[
\frac{C_m}{M_1} = 0.25
\]

\[
J = 0.4 \rightarrow \text{shroud ratio}
\]

\[
\frac{r_3/r_2} = 0.7
\]

\[
\frac{w_3/w_2} = 2
\]

\[
\begin{align*}
\rho_3 & = 12 \\
\omega & = 250 \text{ rad/sec} \\
T_{01} & = 1050 \text{ K} \\
\omega & = 4.1 \text{ rad/sec} \\
\omega_3 & = 0.31 \\
C_p & = 1150 \text{ J/kgK} \\
k & = 1.333
\end{align*}
\]

\[
\begin{align*}
\dot{m} & = \rho_3 C_m A_3 = \left( \frac{\rho_3}{2 \pi} \right) \left( \frac{C_m M_2}{U_2} \right) \pi \left( \frac{r_3}{r_1} \right)^2 \left( 1 - \sqrt{\frac{A_2}{A_1}} \right)
\end{align*}
\]

\[
\begin{align*}
T_{03} & = T_{01} (1 - \delta) = 1050 \times 0.8 = 840 \text{ K} \\
T_3 & = T_{03} - \frac{C_m^2}{2C_p} = T_{03} - \left( \frac{C_m}{U_2} \right)^2 \frac{V^2}{2C_p}
\end{align*}
\]

\[
= 840 - 0.25^2 \times 5.381^2 \times \frac{2 \times 150.7}{2} = 832.1 \text{ K}
\]

\[
\begin{align*}
A & = \left( \frac{10^5}{337.38 L} \right) \times 0.25 \times 5.381 \times 0.9^2 \times \pi \left( 1 - 0.4^2 \right) L_2^2
\end{align*}
\]

\[
\begin{align*}
\dot{m} & = \rho_3 C_m A_3 \\
\text{and} \ A_2 & = 2 \pi r_2 b_2 = 4 \pi r_2^2 \left( b_2/d_2 \right)
\end{align*}
\]

\[
\begin{align*}
c_0 & = S C_p T_{01}/U_2 = 448.9 \text{ m/s} \\
c_m & = c_0^2 / \sin d_2 = 135.4 \text{ m/s} \\
c_2 & = c_0^2 / \sin d_2 = 468.8 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
T_2 & = T_{02} - \frac{c_2^2}{k p_2} = 954.5 \text{ K} \\
h_{02} - h_2 & = \frac{1}{2} c_2^2 \\
\text{and} \ D_n & = \frac{h_2 - h_2}{\frac{1}{2} c_2^2}
\end{align*}
\]

\[
\begin{align*}
\frac{T_{02} - T_{21}}{T_{02}} & = \frac{c_2^2 (1 + D_n)}{2 C_p T_{02}} = 0.0964 \text{ K} \\
\frac{T_{03}}{T_{01}} & = \left( \frac{\rho_2}{\rho_0} \right)^{k/2} = 0.90335 \\
\left( \frac{\rho_2}{\rho_0} \right) & = 0.66652
\end{align*}
\]

\[
\begin{align*}
\frac{b_2}{D_2} & = \frac{4}{W_1} \left( \frac{D_2}{R_1} \right) \left( \frac{\dot{m}}{C_m R_2^2} \right) = 0.0566
\end{align*}
\]
Hydraulic turbines

The American engineer James B. Francis designed the first radial-inflow hydraulic turbine that became widely used, gave excellent results, and was highly regarded. In its original form it was used for heads of between 10 and 100 m.

The Pelton wheel turbine, named after its American inventor, Lester A. Pelton, was brought into use in the second half of the nineteenth century. This is an impulse turbine in which water is piped at high pressure to a nozzle where it expands completely to atmospheric pressure. The emerging jet impacts onto the blades (or buckets) of the turbine, which produce the required torque and power output.

The increasing need for more power during the early years of the twentieth century also led to the invention of a turbine suitable for small heads of water, i.e., 3 to 9 m, in river locations where a dam could be built. In 1913 Viktor Kaplan revealed his idea of the propeller (or Kaplan) turbine, which acts like a ship’s propeller but in reverse. At a later date Kaplan improved his turbine by means of swiveling blades, which improved the efficiency of the turbine appropriate to the available flow rate and head.
Hydraulic turbines

Power Specific Speed

\[ \Omega_{sp} = \frac{\Omega \sqrt{P/\rho}}{(gH_E)^{\frac{2}{3}}} \]

where \( P \) is the power delivered by the shaft, \( \rho \) is the density of water, \( H_E \) is the effective head at turbine entry, and \( \Omega \) is the rotational speed in radians per second. It is remarkable that the efficiency of the multi-stage Pelton turbine has now reached 92.5% at \( \Omega_{sp} \approx 0.2 \) and that the Francis turbine can achieve an efficiency of 95 to 96% at an \( \Omega_{sp} \approx 1.0 \) to 2.0.

Efficiencies of common hydraulic turbines

<table>
<thead>
<tr>
<th>Specific speed (rad)</th>
<th>Pelton Turbine</th>
<th>Francis Turbine</th>
<th>Kaplan Turbine</th>
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<tr>
<td>Head (m)</td>
<td>100–1770</td>
<td>20–900</td>
<td>6–70</td>
</tr>
<tr>
<td>Maximum power (MW)</td>
<td>500</td>
<td>600</td>
<td>300</td>
</tr>
<tr>
<td>Optimum efficiency (%)</td>
<td>90</td>
<td>95</td>
<td>94</td>
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<tr>
<td>Regulation method</td>
<td>Needle valve and deflector plate</td>
<td>Stagger angle of guide vanes</td>
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Hydraulic turbines

Courtesy of Sulzer Hydro Ltd.
Hydraulic turbines – The Pelton Turbine

A Multi-nozzle Pelton turbine
Hydraulic turbines – The Pelton Turbine

the specific work done by the water \( \Delta W = U_1 c_{\theta 1} - U_2 c_{\theta 2} \).

For the Pelton turbine, \( U_1 = U_2 = U \), \( c_{\theta 1} = c_1 \) so we get

\[
\Delta W = U[U + w_1 - (U + w_2 \cos \beta_2)] = U(w_1 - w_2 \cos \beta_2).
\]

\[c_{\theta 2} = U + w_2 \cos \beta_2.\]

The effect of friction on the fluid flowing inside the bucket will cause the relative velocity at outlet to be less than the value at inlet. Writing \( w_2 = k w_1 \), where \( k < 1 \),

\[
\Delta W = U w_1 (1 - k \cos \beta_2) = U (c_1 - U)(1 - k \cos \beta_2).
\]

An efficiency \( \eta_R \) for the runner can be defined as

\[
\eta_R = \Delta W \sqrt{\left(\frac{1}{2} c_1^2\right)} = 2U (c_1 - U)(1 - k \cos \beta_2)/c_1^2.
\]

\[\eta_R = 2v(1 - v)(1 - k \cos \beta_2),\]

where the blade speed to jet speed ratio, \( v = U/c_1 \).

the maximum efficiency of the runner occurs when \( v = 0.5 \)

\[\eta_{R_{\text{max}}} = (1 - k \cos \beta_2)\]
Hydraulic turbines – The Pelton Turbine

A common Pelton Turbine efficiency trend with the blade – jet speed ratio
Hydraulic turbines – The Pelton Turbine

A constant level reservoir at an elevation $z_R$ (above sea level) and flows via a pressure tunnel to the penstock head, down the penstock to the turbine nozzles emerging onto the buckets as a high speed jet. To reduce the deleterious effects of large pressure surges, a surge tank is connected to the flow close to the penstock head, which acts so as to damp out transients. The elevation of the nozzles is $z_N$ and the gross head, $H_G = z_R - z_N$.

Pelton Turbine Hydroelectric Scheme
Hydraulic turbines – The Pelton Turbine

Regulating the speed of a pelton runner
Hydraulic turbines – The Pelton Turbine

Sizing the penstock with Darcy’s Equation

\[ H_f = \frac{2flV^2}{gd} \]

- \( f \): friction factor (recall Moody diagram)
- \( l \): Length of the pipe
- \( d \): pipe diameter
- \( V \): Average fluid velocity in the pipe

Substituting for the velocity, \( V = 4Q/(\pi d^2) \), we get

\[ H_f = \left( \frac{32fl}{\pi^2 g} \right) \frac{Q^2}{d^5}. \]
Hydraulic turbines – The Pelton Turbine

Moody Diagram

Transition Region

Laminar Flow

Friction Factor

Material

Concrete coarse
Concrete, fine smooth
Brick, rough
Glares, Plastic Pipes
Steel, cast
Steel, smooth
Steel, structural

Reynolds Number, \( Re = \frac{\nu D_l}{\mu} \)

Friction Factor = \( \frac{2\varepsilon}{\rho V^2 A_1} \)
Hydraulic turbines – The Pelton Turbine

Example 1

Water is supplied to a turbine at the rate $Q = 2.272 \, \text{m}^3/\text{s}$ by a single penstock 300 m long. The allowable head loss due to friction in the pipe amounts to 20 m. Determine the diameter of the pipe if the friction factor $f = 0.01$.

$$d^5 = \frac{32 \, fl \left( \frac{Q}{\pi} \right)^2}{gH_f} = \frac{32 \times 0.01 \times 300 \left( \frac{2.272}{\pi} \right)^2}{9.81 \times 20} = 0.2559.$$

$d = 0.7614 \, \text{m}$
Hydraulic turbines – The Pelton Turbine

Example 2

A Pelton turbine is driven by two jets, generating 4.0 MW at 375 rev/min. The effective head at the nozzles is 200 m of water and the nozzle velocity coefficient, $K_N = 0.98$. The axes of the jets are tangent to a circle 1.5 m in diameter. The relative velocity of the flow across the buckets is decreased by 15% and the water is deflected through an angle of 165°.

Neglecting bearing and windage losses, determine

(i) the runner efficiency;
(ii) the diameter of each jet;
(iii) the power specific speed.

(i) The blade speed is

$$U = \Omega r = (375 \times \pi / 30) \times 1.5 / 2 = 39.27 \times 1.5 / 2 = 29.45 \text{ m/s}.$$  

The jet speed is

$$c_1 = K_N \sqrt{2gH_E} = 0.98 \times \sqrt{2 \times 9.81 \times 200} = 61.39 \text{ m/s}.$$  

Therefore, $v = U / c_1 = 0.4798$.

The efficiency of the runner is obtained

$$\eta_R = 2 \times 0.4798 \times (1 - 0.4798) (1 - 0.85 \times \cos 165°) = 0.9090.$$  

(ii) The “theoretical” power is $P_t = P / \eta_R = 4.0 / 0.9090 = 4.40 \text{ MW}$, where $P_t = \rho gQH_E$. Therefore,

$$Q = P_t / (\rho gH_E) = 4.40 \times 10^6 / (9810 \times 200) = 2.243 \text{ m}^3 / \text{s}.$$  

Each jet must have a flow area of

$$A_j = \frac{Q}{2c_1} = \frac{2.243}{2 \times 61.39} = 0.01827 \text{ m}^2.$$  

Therefore, $d_j = 0.5125 \text{ m}$.

(iii) the power specific speed is

$$\Omega_{sp} = 39.27 \times \left(\frac{4.0 \times 10^6}{10^3}\right)^{1/2} / (9.81 \times 200)^{1/2} = 0.190 \text{ rad}.$$
Hydraulic turbines – Reaction Turbines

(i) only part of the overall pressure drop has occurred up to turbine entry, the remaining pressure drop takes place in the turbine itself;
(ii) the flow completely fills all of the passages in the runner, unlike the Pelton turbine where, for each jet, only one or two of the buckets at a time are in contact with the water;
(iii) pivotal guide vanes are used to control and direct the flow;
(iv) a draft tube is normally added on to the turbine exit; this is considered as an integral part of the turbine.
Hydraulic turbines – Francis Turbine

Inlet-exit velocity triangles

Comparison of velocity triangles

Comparison of efficiencies
Hydraulic turbines – Francis Turbine

Euler’s turbine equation  \[ \Delta W = U_2 c_{\theta 2} - U_3 c_{\theta 3} \]

If the flow at runner exit is without swirl then the equation reduces to  \[ \Delta W = U_2 c_{\theta 2} \]

At entry to the runner the energy available is equal to  \[ g(H_E - \Delta H_N) = \frac{p_2 - p_a}{\rho} + \frac{1}{2} c_2^2 + g z_2. \]

\( \Delta H_N \) is the loss of head due to friction in the volute and guide vanes and \( p_2 \) is the absolute static pressure at inlet to the runner.

At runner outlet the energy in the water is further reduced by the amount of specific work \( \Delta W \) and by friction work in the runner, \( g \Delta H_R \) and this remaining energy equals the sum of the pressure potential and kinetic energies:

\[ g(H_E - \Delta H_N - \Delta H_R) - \Delta W = \frac{1}{2} c_3^2 + p_3/\rho - p_a/\rho + g z_3 \]

where \( p_3 \) is the absolute static pressure at runner exit.

The specific work is  \[ \Delta W = (p_{02} - p_{03})/\rho - g \Delta H_R + g(z_2 - z_3) \]
Hydraulic turbines – Francis Turbine

The energy equation between the exit of the runner and the tailrace

\[\frac{p_3}{\rho} + \frac{1}{2}c_3^2 + gz_3 - g\Delta H_{DT} = \frac{1}{2}c_4^2 + p_a/\rho\]

where \(\Delta H_{DT}\) is the loss in head in the draft tube and \(c_4\) is the flow exit velocity.

The hydraulic efficiency is defined by

\[\eta_h = \frac{\Delta W}{gH_E} = \frac{U_2c_{\theta 2} - U_3c_{\theta 3}}{gH_E}\]

and, whenever \(c_{\theta 3} = 0\),

\[\eta_H = \frac{U_2c_{\theta 2}}{gH_E}\]

The overall efficiency is given by \(\eta_o = \eta_m\eta_H\)
Hydraulic turbines – Francis Turbine

Example 3

In a vertical-shaft Francis turbine the available head at the inlet flange is 150 m of water and the vertical distance between the runner and the tailrace is 2.0 m. The runner tip speed is 35 m/s, the meridional velocity of the water through the runner is constant at 10.5 m/s, the flow leaves the runner without whirl and the velocity at exit from the draft tube is 3.5 m/s.

The hydraulic losses for the turbine are as follows:

$$\Delta H_N = 6.0 \text{ m}, \quad \Delta H_R = 10 \text{ m}, \quad \Delta H_{DT} = 1.0 \text{ m}.$$  

Determine

(i) the specific work, $\Delta W$, and the hydraulic efficiency, $\eta_h$, of the turbine;
(ii) the absolute velocity, $c_2$, at runner entry;
(iii) the pressure head (relative to the tailrace) at inlet to and exit from the runner;
(iv) the absolute and relative flow angles at runner inlet;
(v) if the flow discharged by the turbine is 20 m$^3$/s and the power specific speed is 0.8 (rad), the speed of rotation and diameter of the runner.

$$\Delta W = g(H_E - \Delta H_N - \Delta H_R - \Delta H_{DT}) - \frac{1}{2}c_4^2$$

$$= 9.81 \times (150 - 6 - 10 - 1) - 3.5^2/2 = 1298.6 \text{ m}^2/\text{s}^2.$$  

The hydraulic efficiency, $\eta_h = \Delta W/(gH_E) = 0.8825$.

As $c_{03} = 0$, then $\Delta W = U_2c_{02}$ and $c_{02} = \Delta W/U_2 = 1298.6/35 = 37.1 \text{ m/s}$, thus,

$$c_2 = \sqrt{c_{02}^2 + c_m^2} = \sqrt{37.1^2 + 10.5^2} = 38.56 \text{ m/s}.$$
Hydraulic turbines – Francis Turbine

\[ H_2 = H_E - \Delta H_N - \frac{c_2^2}{2g} = 150 - 6 - \frac{38.56^2}{2 \times 9.81} = 68.22 \text{ m} \]

\[ H_3 = \frac{(p_3 - p_a)}{(\rho g)} = \frac{(c_4^2 - c_3^2)}{(2g)} + \Delta H_{DT} - z_3 = \frac{(3.5^2 - 10.5^2)}{(2 \times 9.81)} + 1 - 2 = -6.0 \text{ m} \]

The flow angles at runner inlet are now obtained as follows:

\[ \alpha_2 = \tan^{-1}\left(\frac{c_{\theta_2}}{c_{r_2}}\right) = \tan^{-1}\left(\frac{37.1}{10.5}\right) = 74.2^\circ \]

\[ \beta_2 = \tan^{-1}\left[\left(\frac{c_{\theta_2} - U_2}{c_{r_2}}\right)\right] = \tan^{-1}\left[\left(\frac{37.1 - 35}{10.5}\right)\right] = 11.31^\circ \]

\[ \Omega = \frac{\Omega_{sp}(gH_E)^{\frac{3}{2}}}{\sqrt{Q\Delta W}} = \frac{0.8 \times 91.14}{\sqrt{20 \times 1298.7}} = 45.24 \text{ rad/s} \]

Thus, the rotational speed \( N = 432 \text{ rev/min} \) and the runner diameter is

\[ D_2 = 2U_2/\Omega = 70/45.24 = 1.547 \text{ m} \]
Hydraulic turbines – Kaplan Turbine

This type of turbine evolved from the need to generate power from much lower pressure heads than are normally employed with the Francis turbine. To satisfy large power demands very large volume flow rates need to be accommodated in the Kaplan turbine, i.e., the product $QH_f$ is large. The overall flow configuration is from radial to axial.
Hydraulic turbines – Kaplan Turbine

Brillant (Canada), 2006 - 1 x 120 MW - Head: 30 m.
Grand Mère (Canada), 2004 - 3 x 77 MW - Head: 24 m.
Qing Shan Dian (China), 1998 - 2 x 20 MW - Head: 29 m.
Terminus Dam (USA), 1995 - 1 x 17 MW - Head: 52 m.
Porto Primavera (Brazil), 1999-2003 - 18 x 101 MW - Head: 10 m.
Buyo (Ivory Coast), 1976 - 3 x 45 MW - Head: 27 m.

Source: gepower.com
Hydraulic turbines – Kaplan Turbine

\[ c_{\theta 2} = \frac{K}{r}, \; c_x = \text{a constant.} \]

\[ \tan \beta_2 = \frac{U}{c_x} - \tan \alpha_2 = \frac{\Omega r}{c_x} - \frac{K}{rc_x}, \]

\[ \tan \beta_3 = \frac{U}{c_x} = \frac{\Omega r}{c_x}. \]
Hydraulic turbines – Kaplan Turbine

Example 4

A small-scale Kaplan turbine has a power output of 8 MW, an available head at turbine entry of 13.4 m, and a rotational speed of 200 rev/min. The inlet guide vanes have a length of 1.6 m and the diameter at the trailing edge surface is 3.1 m. The runner diameter is 2.9 m and the hub-tip ratio is 0.4.

Assuming the hydraulic efficiency is 92% and the runner design is “free-vortex,” determine

(i) the radial and tangential components of velocity at exit from the guide vanes;
(ii) the component of axial velocity at the runner;
(iii) the absolute and relative flow angles upstream and downstream of the runner at the hub, mid-radius, and tip.

As \( P = \eta_H \rho g Q H_E \), then the volume flow rate is

\[
Q = \frac{P}{(\eta_H \rho g H_E)} = \frac{8 \times 10^6}{(0.92 \times 9810 \times 13.4)} = 66.15 \text{ m/s}^2
\]

Therefore,

\[
c_{11} = \frac{Q}{2 \pi r_1 L} = \frac{66.15}{(2 \pi \times 1.55 \times 1.6)} = 4.245 \text{ m/s},
\]

\[
c_{12} = \frac{4Q}{\pi D_2^2 (1 - \nu^2)} = \frac{4 \times 66.15}{(\pi \times 2.9^2 \times 0.84)} = 11.922 \text{ m/s}.
\]

As the specific work done is \( \Delta W = U_2 c_{02} \) and \( \eta_H = \Delta W/(g H_E) \), then at the tip

\[
c_{02} = \frac{\eta_H g H_E}{U_2} = \frac{0.92 \times 9.81 \times 13.4}{30.37} = 3.892 \text{ m/s},
\]
Hydraulic turbines – Kaplan Turbine

where the blade tip speed is \( U_2 = \Omega D_2/2 = (200 \times \pi/30) \times 2.9/2 = 30.37 \text{ m/s}, \)

\[ c_{\theta 1} = c_{\theta 2} r_2/r_1 = 3.892 \times 1.45/1.55 = 3.725 \text{ m/s}^2, \]

\[ \alpha_1 = \tan^{-1}\left(\frac{c_{\theta 1}}{c_{r 1}}\right) = \tan^{-1}\left(\frac{3.725}{4.245}\right) = 41.26^\circ. \]

\[ \alpha_2 = \tan^{-1}\left(\frac{c_{\theta 2}}{c_{r 2}}\right) = \tan^{-1}\left(\frac{c_{\theta 1} r_1}{c_{r 2} r}\right), \]

\[ \beta_2 = \tan^{-1}\left(\frac{\Omega r}{c_{r 2}} - \tan \alpha_2\right) = \tan^{-1}\left(\frac{U_2 r}{c_{r 2}} r_1 - \tan \alpha_2\right), \]

\[ \beta_3 = \tan^{-1}\left(\frac{U}{c_{r 2}}\right) = \tan^{-1}\left(\frac{U_2 r}{c_{r 2}} r_1\right). \]

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<th>0.4</th>
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<td>( \theta_2 ) (m/s)</td>
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<td>( \tan \alpha_2 )</td>
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<td>( \alpha_2 ) (deg)</td>
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<td>( \beta_2 ) (deg)</td>
<td>10.43</td>
<td>52.56</td>
<td>65.69</td>
</tr>
<tr>
<td>( \beta_3 ) (deg)</td>
<td>45.54</td>
<td>60.72</td>
<td>68.57</td>
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</table>
Hydraulic turbines – Bulb Turbine
Hydraulic turbines – Bulb Turbine

Jirau and Santo Antonio (Brazil), 2012/2013
10 x 76.65 MW and 19 x 75.5 MW

Chang Zhou and Qiao Gong (China), 2008
3 x 42 MW - Head: 10 m. and 4 x 57 MW - Head: 14 m.

Ourhinos (Brazil), 2004
3 x 15 MW - Head: 11 m.

Paldang (South Korea), 1973
4 x 21 MW - Head: 12 m.

St Egrève (France), 1990
2 x 23 MW turbine + generator - Head: 12 m.

Source: ge-power.com
Effect of size on turbine efficiencies

To illustrate this consider a family of turbomachines where the loading term, \( \psi = gH/N^2D^2 \), is the same and the Reynolds number, \( \text{Re} = ND^2/\nu \), is the same for every size of machine, then

\[
\psi \text{Re}^2 = \frac{gH}{N^2D^2} \times \frac{N^2D^4}{\nu^2} = \frac{gHD^2}{\nu^2}
\]

must be the same for the whole family. Thus, for a given fluid (\( \nu \) is a constant), a reduction in size \( D \) must be followed by an increase in the head \( H \). A turbine model of one eighth the size of a prototype would need to be tested with a head 64 times that required by the prototype! Fortunately, the effect on the model efficiency caused by changing the Reynolds number is not large. In practice, models are normally tested at conveniently low heads and an empirical correction is applied to the efficiency.

With model testing other factors affect the results. Exact geometric similarity cannot be achieved for the following reasons:

(i) the blades in the model will probably be thicker than in the prototype;
(ii) the relative surface roughness for the model blades will be greater;
(iii) leakage losses around the blade tips of the model will be relatively greater as a result of increased relative tip clearances.

\[
\frac{1 - \eta_p}{1 - \eta_m} = \left( \frac{D_m}{D_p} \right)^n, \quad n = 0.25
\]
Effect of size on turbine efficiencies

Example 5

A model of a Francis turbine is built to a scale of one fifth of full size and when tested it developed a power output of 3 kW under a head of 1.8 m of water, at a rotational speed of 360 rev/min and a flow rate of 0.215 m$^3$/s. Estimate the speed, flow rate, and power of the full-scale turbine when working under dynamically similar conditions with a head of 60 m of water.

By making a suitable correction for scale effects, determine the efficiency and the power of the full-size turbine. Use Moody’s formula and assume $n = 0.25$.

From the group $\psi = gH/(ND)^2$ we get

$$N_p = N_m(D_n/D_p)(H_p/H_m)^{0.5} = (360/5)(60/1.8)^{0.5} = 415.7 \text{ rev/min}.$$  

From the group $\phi = Q/(ND^3)$ we get

$$Q_p = Q_m(N_p/N_m)(D_p/D_m)^3 = 0.215 \times (415.7/360) \times 5^3 = 31.03 \text{ m}^3/\text{s}.$$  

Lastly, from the group $\dot{P} = P/\left(\rho N^3 D^3\right)$ we get

$$P_p = P_m(N_p/N_m)^5 (D_p/D_m)^5 = 3 \times (415.7)^3 \times 5^5 = 14,430 \text{ kW} = 14.43 \text{ MW}.$$  

This result has still to be corrected to allow for scale effects. First we must calculate the efficiency of the model turbine. The efficiency is found from

$$\eta_m = P/\left(\rho Q g H\right) = 3 \times 10^3/(10^3 \times 0.215 \times 9.81 \times 1.8) = 0.79.$$  

Using Moody’s formula the efficiency of the prototype is determined:

$$(1 - \eta_p) = (1 - \eta_m) \times 0.2^{0.25} = 0.21 \times 0.6687,$$

hence,

$$\eta_p = 0.8596.$$  

The corresponding power is found by an adjustment of the original power obtained under dynamically similar conditions, i.e.,

$$\text{corrected } P_p = 14.43 \times 0.8596/0.79 = 15.7 \text{ MW}.$$