Introduction

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Research Fields: Thermal system design, renewable energy, hydrogen production, Energy Storage
Introduction

Marking Scheme
Midterm – 40%
Project – 20% (added to final exam - elective)
Final Exam – 60%

Sources
Basic Concepts in Turbomachinery (Ingram, 2009)
Turbomachinery Performance Analysis (Levis, 1996)
Fluid Mechanics and thermodynamics of Turbomachinery (Dixon and Hall, 2010)

Course Content
Week 1: Introduction and Basic Principles
Week 2: Relative and Absolute Motion
Week 3: Turbomachine Operation
Week 4: Basic Concepts for Fluid Motion
Week 5: Dimensionless Parameters in Turbomachinery
Week 6: Efficiency and Reaction for Turbomachinery
Week 7-9: Axial flow Machines
Week 9-12: Hydraulic Turbines
Week 11-13: Analysis of Pumps
Week 14: Project presentations
Introduction to Turbomachinery

What is a turbomachine?

- Turbomachine is a device exchanging energy with a fluid by either absorbing from or extracting energy to the fluid.
- The energy extracting devices are called *Turbine*, while energy absorbing devices can be called *pump, blower, fan, and compressor.*
Introduction to Turbomachinery

(a) Single stage axial flow compressor or pump

(b) Mixed flow pump

(c) Centrifugal compressor or pump

(d) Francis turbine (mixed flow type)

(e) Kaplan turbine

(f) Pelton wheel

Some Applications of Turbomachinery
Introduction to Turbomachinery

Classification of Turbomachines

- Energy Extracting – Energy Absorbing
- Flow Direction: Axial, radial, mixed
- Fluid type: Compressible, incompressible
- Impulse, reaction (For hydro turbines)
Introduction to Turbomachinery

Energy Extracting Devices

*Gas Turbine: Energy production is accomplished by extracting energy from a compressible gas such as air, helium, or CO₂.*
Introduction to Turbomachinery

Energy Extracting Devices

*Steam Turbine*: Energy production is accomplished by extracting energy from superheated high pressure steam. Many other liquid working fluids are also in use.
Introduction to Turbomachinery

Energy Extracting Devices

*Wind Turbine: Converts kinetic energy of air into mechanical energy.*
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Energy Extracting Devices

*Hydro Turbine:* Hydoturbines are used to convert the potential energy of water into mechanical energy.
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Energy Absorbing Devices

*Pumps:* Absorbs energy to increase the pressure of a liquid.
Introduction to Turbomachinery

Energy Absorbing Devices

*Fans: Low pressure increase of high flow rate gases.*
Introduction to Turbomachinery

Energy Absorbing Devices

*Blowers: Medium pressure increase of medium flow rate gases.*
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Energy Absorbing Devices

Compressors: High pressure increase of low flow rate gases
Introduction to Turbomachinery

Positive displacement vs dynamic turbomachines

Fluid Machinery

Positive Displacement
- Working fluid is confined within a boundary.
- Energy transfer is by volume changes due to the movement of the boundary.

Dynamic
- Working fluid is not confined within a boundary.
- Energy transfer is by dynamic effects of the rotor on the fluid stream.

Courtesy of slidesharecdn.com
### Introduction to Turbomachinery

**Fluid Type**

*Compressible: (Fans, blowers, compressors, gas turbines)*

*Incompressible: (hydro turbines)*

<table>
<thead>
<tr>
<th><strong>IMPULSE TURBINE VS REACTION TURBINE</strong></th>
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</thead>
<tbody>
<tr>
<td><strong>Impulse turbine</strong></td>
<td><strong>Reaction turbine</strong></td>
<td></td>
</tr>
<tr>
<td>The steam completely expands in the nozzle and its pressure remains constant during its flow through the blade passages</td>
<td>The steam expands partially in the nozzle and further expansion takes place in the rotor blades</td>
<td></td>
</tr>
<tr>
<td>The relative velocity of steam passing over the blade remains constant in the absence of friction</td>
<td>The relative velocity of steam passing over the blade increases as the steam expands while passing over the blade</td>
<td></td>
</tr>
<tr>
<td>Blades are symmetrical</td>
<td>Blades are asymmetrical</td>
<td></td>
</tr>
<tr>
<td>The pressure on both ends of the moving blade is same</td>
<td>The pressure on both ends of the moving blade is different</td>
<td></td>
</tr>
<tr>
<td>For the same power developed, as pressure drop is more, the number of stages required are less</td>
<td>For the same power developed, as pressure drop is small, the number of stages required are more</td>
<td></td>
</tr>
<tr>
<td>The blade efficiency curve is less flat</td>
<td>The blade efficiency curve is more flat</td>
<td></td>
</tr>
<tr>
<td>The steam velocity is very high and therefore the speed of turbine is high.</td>
<td>The steam velocity is not very high and therefore the speed of turbine is low.</td>
<td></td>
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</tbody>
</table>
Introduction to Turbomachinery

1. Coordinate System

Since there are stationary and rotating blades in turbomachines, they tend to form a cylindrical form, represented in three directions;

1. Axial
2. Radial
3. Tangential (Circumferential - \( r\theta \))
Introduction to Turbomachinery

1. Coordinate System

The Velocity at the meridional direction is:

\[ c_m = \sqrt{c_x^2 + c_r^2} \]

Where \( x \) and \( r \) stand for axial and radial.

NOTE: In purely axial flow machines \( C_r = 0 \), and in purely radial flow machines \( C_x = 0 \).
Introduction to Turbomachinery

1. Coordinate System

Total flow velocity is calculated based on below view as

$$ c = \sqrt{c_x^2 + c_r^2 + c_\theta^2} = \sqrt{c_m^2 + c_\theta^2} $$

The swirl (tangential) angle is

$$ \alpha = \tan^{-1}(c_\theta/c_m) $$

Relative Velocities

Relative Velocity

$$ w_\theta = c_\theta - U $$

Relative Flow Angle

$$ \beta = \tan^{-1}(w_\theta/c_m) $$

Combining i, ii, and iii:

$$ \tan \beta = \tan \alpha - U/c_m $$
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

2.1. Continuity Equation (Conservation of mass principle)

\[ \dot{m} = \rho_1 c_1 A_{n1} = \rho_2 c_2 A_{n2} \]

2.2. Conservation of Energy (1st law of thermodynamics)

Stagnation enthalpy;

\[ h_0 = h + \frac{1}{2} c^2 \]

if \( gz = 0 \);

\[ \dot{Q} - \dot{W}_x = \dot{m}(h_{02} - h_{01}) \]

For work producing machines

\[ \dot{W}_x = \dot{W}_t = \dot{m}(h_{01} - h_{02}) \]

For work consuming machines

\[ \dot{W}_c = -\dot{W}_x = \dot{m}(h_{02} - h_{01}) \]
### Introduction to Turbomachinery

#### 2. Fundamental Laws used in Turbomachinery

##### 2.3. Conservation of Momentum (Newton's Second Law of Motion)

- For a steady flow process:

- Here, $pA$ is the **pressure contribution**, where it is cancelled when there is rotational symmetry. Using this basic rule one can determine the angular momentum as

- The Euler work equation is:

\[
\tau_A = \dot{m}(r_2 c_{\theta 2} - r_1 c_{\theta 1})
\]

\[
\tau_A \Omega = \dot{m}(U_2 c_{\theta 2} - U_1 c_{\theta 1})
\]

where \( U = \Omega r \)

The Euler Pump equation:

\[
\Delta W_c = \frac{\dot{W}_c}{\dot{m}} = \frac{\tau_A \Omega}{\dot{m}} = U_2 c_{\theta 2} - U_1 c_{\theta 1} > 0
\]

The Euler Turbine equation:

\[
\Delta W_t = \frac{\dot{W}_t}{\dot{m}} = U_1 c_{\theta 1} - U_2 c_{\theta 2} > 0
\]
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

Writing the Euler Equation in the energy equation for an adiabatic turbine or pump system \( Q=0 \)

\[
\Delta W_x = (h_{01} - h_{02}) = U_1 c_{\theta 1} - U_2 c_{\theta 2}
\]

\[
\Delta h_0 = \Delta (Uc_\theta)
\]

NOTE: Frictional losses are not included in the Euler Equation.

2.4. Rothalpy

An important property for fluid flow in rotating systems is called rothalpy \( (I) \)

\[
I = h_0 - Uc_\theta \quad \text{and} \quad I = h + \frac{1}{2} c^2 - Uc_\theta
\]

Writing the velocity \( (c) \), in terms of relative velocities:

\[
I = h + \frac{1}{2} (w^2 + U^2 + 2Uw_\theta) - U(w_\theta + U)
\]

, simplifying;

Defining a new RELATIVE stagnation enthalpy;

Redefining the Rothalpy:

\[
I = h_{0,rel} - \frac{1}{2} U^2
\]

\[
h_{0,rel} = h + \frac{1}{2} w^2
\]

\[
h + \frac{1}{2} w^2 - \frac{1}{2} U^2
\]
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

2.6. Second Law of Thermodynamics

The Clasius Inequality:
\[ \int \frac{dQ}{T} \leq 0 \]

For a reversible cyclic process:
\[ \int \frac{dQ_R}{T} = 0 \]

Entropy change of a state is,
\[ dS = md_s = \frac{dQ_R}{T} \]

that we can evaluate the isentropic process when the process is reversible and adiabatic (hence isentropic).

Here we can re-write the above definition as
\[ dQ = dQ_R = mTds \]

and using the first law of thermodynamics:
\[ dQ - dW = dh = du + pdv \]

\[ Tds = du + pdv \]

and
\[ Tds = dh - vdp. \]
**Introduction to Turbomachinery**

From the Energy Balance,
\[
\Delta Q_r = (\dot{S}_{\text{tot}} + \dot{S}_{\text{wp}}) \frac{\Delta W_{\text{tot}}}{T_r}
\]

Using Carnot's statement,
\[
\Delta Q_r = \frac{\Delta Q}{T_r} \Rightarrow \Delta Q_r = \Delta W_{\text{tot}} \frac{\Delta Q}{T_r}
\]

\[
\Rightarrow \frac{\Delta Q}{T_r} = \Delta W_{\text{tot}} \Rightarrow \Delta W_{\text{tot}} = \frac{\Delta Q}{T_r}
\]

Considering Kelvin-Plank statement “No cyclic system can produce a net amount of work by heat transfer with one heat reservoir.” Therefore \(\Delta W_{\text{tot}}\) is either ‘0’ or negative. Since \(T_r\) is definitely a positive value:
\[
\frac{\Delta Q}{T_r} \leq 0
\]

\[\text{Now consider a process:}\]

1 → 2: irreversible
2 → 1: reversible
\[
\int_1^2 \frac{\Delta Q}{T_r} = \int_1^2 \left(\frac{\Delta Q}{T_{\text{irr}}} + \frac{\Delta Q}{T_{\text{rev}}}\right) \leq 0
\]

\[
\int_1^2 \frac{\Delta Q}{T_{\text{irr}}} + \int_1^2 ds \leq 0 \Rightarrow ds > (\frac{\Delta Q}{T_{\text{irr}}})_{\text{rev}}
\]

\[\text{When the process is adiabatic (} \Delta Q = 0\text{)} \Rightarrow ds \

\]

Using Gibbs (TdS) relations:
\[
\Delta Q = \Delta W + dE \rightarrow 1^{\text{st}} \text{law} \quad \quad (i)
\]
\[
\Delta S = (\frac{\Delta Q}{T})_{\text{rev}} \quad \quad (ii) \quad dU = \Delta W + \frac{\Delta Q}{\text{rev}} \quad \quad (iii)
\]

\[\Rightarrow Tds = PdV + dU \rightarrow \text{(pc and be are neglected)}
\]

and \(dh = dU + PdV\) \(\text{chemical rule}\)
\[\Rightarrow
\]

\[
Tds = dh + PdV \quad --\rightarrow \text{control mass}
\]
\[
Tds = dh - VdP \quad --\rightarrow \text{control volume}
\]

\[\text{* Recall entropy change } ds \text{ in solids,}\]

\[\text{ideal gases that are leading us to isentropic relations!!}
\]

\[
\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}
\]
\[
\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k}{k-1}}
\]
\[
\left(\frac{V_1}{V_2}\right)^{k} = \left(\frac{P_2}{P_1}\right)
\]

\[s_1 = s_2\]
2. Fundamental Laws used in Turbomachinery

2.5. Bernoulli’s Equation

Writing an energy balance for a flow, where there is no heat transfer or power production/consumption, one obtains:

\[
(h_2 - h_1) + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1) = 0
\]

Applying for a differential control volume:

\[
dh + cdc + gdz = 0 \quad \text{(where enthalpy is } \hspace{1em} dh = \nu dp = dp/\rho)\]

When the process is isentropic \( Tds = dh - \nu dp \), one obtains Euler’s motion equation:

\[
\frac{1}{\rho} dp + cdc + gdz = 0
\]

Integrating this equation into stream direction, Bernoulli equation is obtained:

\[
\int_{1}^{2} \frac{1}{\rho} dp + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1) = 0
\]

When the flow is incompressible, density does not change, thus the equation becomes:

\[
\frac{1}{\rho} (p_0 - p_01) + g(z_2 - z_1) = 0
\]

where \( p_0 = p + \frac{1}{2} \rho c^2 \) and \( p_0 \) is called as stagnation pressure.

For hydraulic turbomachines head is defined as \( H = z + p_0/(\rho g) \) thus the equation takes the form \( H_2 - H_1 = 0 \).

**NOTE:** If the pressure and density change is negligibly small, than the stagnation pressures at inlet and outlet conditions are equal to each other (This is applied to compressible isentropic processes)
2017 Midterm Q: Using the first law of thermodynamics, show the Bernoulli equation yields to \( H_{in} = H_{out} \) for hydraulic turbomachines.

\[
E_{in} + Q_{in} + W_{in} = E_{out} + Q_{out} + W_{out} \\
\dot{m}(h_{in} + \frac{v_{in}^2}{2} + g_z z_{in}) + Q_{in} + W_{in} = \dot{m}(h_{out} + \frac{v_{out}^2}{2} + g_z z_{out}) + Q_{out} + W_{out}
\]

Simdi bu noktada isi transferinin ve herhangi bir g\"u\"s \"uretimi\" yada \"t\"ek\"etim olmadigini var sayalim; \((Q = 0, W = 0)\) oz zaman den klemi:

\[
(h_2 - h_1) + \frac{1}{2} (c_2^2 - c_1^2) + g(z_2 - z_1) = 0
\]

Diferansiyel olarak ak\"if\"tan bir kesit alirsak:

\[
dh + cdc + g dz = 0
\]

\[
 dh = \nu dp = \frac{dp}{\rho}
\]

İzantropik proses ise:\n
\[
Tds = dh - \nu dp \text{ olur. Burdan Euler hareket den klemi;}
\]
\[ \frac{1}{f}.d\rho + cdc + gdz = 0 \rightarrow \text{Denklem integre edilirse;} \]
\[ \int \frac{1}{f} d\rho + \frac{1}{2} (c_2^2 - c_1^2) + g(z_2 - z_1) = 0 \rightarrow \text{Burada sıvı tekerlemesi \& \& yığınluk değişimi olması gerekir; } \int \frac{1}{f} (p_2 - p_1) + g(z_2 - z_1) = 0 \]

Burada \( p_0 = p + \frac{1}{2} fc^2 \rightarrow p_0 \) durma basıncıdır.

Hidrolik turbinatik müh. i. \( H = z + (\frac{p_0}{\gamma}) \) Buradandadendenkle

\[ H_2 - H_1 = 0 \rightarrow H_{\text{lin}} = H_{\text{out}} \]
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

2.7. Compressible flow relations

→ For a perfect gas, the Mach # can be written as, $M = \frac{c}{a} = \frac{c}{\sqrt{\gamma RT}}$. Here $a$ is the speed of sound, $R$, $T$ and $\gamma$ are universal gas constant, temperature in (K), and specific heat ratio, respectively.

→ Above 0.3 Mach #, the flow is taken as compressible, therefore fluid density is no more constant.

→ With the stagnation enthalpy definition, for a compressible fluid:

$C_p T_0 = C_p T + \frac{c^2}{2} = C_p T + \frac{M^2 \gamma RT}{2}$  \hspace{1cm} (i)

Knowing that: $h = u + \frac{1}{2} v^2$

$dh = du + d(RT)$

$C_p dT = C_v dT + RdT$

$C_p = C_v + R$

→ and $\frac{C_p}{C_v} = \gamma$, one gets $\gamma - 1 = \frac{R}{C_v}$

$\gamma R = (\gamma - 1) C_P$ \hspace{1cm} (ii)

$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$ \hspace{1cm} (iii)

Replacing (ii) into (i) one obtains relation between static and stagnation temperatures:

Replacing (ii) into (i) one obtains relation between static and stagnation pressures:

Integrating one obtains the relation between static and stagnation pressures:

$\frac{p_0}{p} = \left( \frac{T_0}{T} \right)^{\gamma/(\gamma-1)} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma/(\gamma-1)}$ \hspace{1cm} (v)
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

Deriving Speed of sound
2. Fundamental Laws used in Turbomachinery

Deriving Speed of sound

From Conservation Of Mass:

\[(pA)_{in} = (pA)_{out}\]

\[pC = (p + dp)(c - dv)\]

\[pC = p - pdv + cdp - dpdW\]

Neglecting the high degree differential term:

\[cdp = pdv\]  \(\text{(2)}\)

Combining (1) and (2):

For a reversible-adiabatic flow:

\[cds = \frac{dp}{c} \rightarrow c^2 = \frac{dp}{cdp}\]

\[C = \sqrt{\frac{dp}{cdp}}\]

For an ideal gas:

\[k = \frac{C_p}{C_v}\]

\[P_v = RT\]

\[dh = dU + PdV = dU + RdT\]

\[CpdT = CvdT + RdT\]

\[R = C_p - C_v\]

From the Gibbs relations:

\[TdS = dU + PdV\]

\[dS = CvdT + \frac{Rf}{V} \frac{dv}{V}\]

\[dS = CvdT + \frac{RdV}{V}\]

\[\rightarrow \text{if the process is isentropic}\]

\[CvdT = -\frac{RdV}{V}\]

\[\int_{T_1}^{T_2} \frac{dT}{T} = \int_{V_1}^{V_2} \left(\frac{-R}{C_v}\right) \frac{dV}{V}\]

\[\ln\left(\frac{T_1}{T_2}\right) = \left(\frac{C_v + C_p}{C_v}\right) \ln\left(\frac{V_1}{V_2}\right) = \left(1 - \frac{k}{k-1}\right) \ln\left(\frac{V_1}{V_2}\right)\]
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

Deriving Speed of sound

\[
\frac{T_i}{T_o} = \left(\frac{V_i}{V_o}\right)^{1-k}
\]

Since \( T_i = \frac{P_iV_i}{R} \)
\( T_o = \frac{P_oV_o}{R} \)

\[
\frac{P_iV_i}{P_oV_o} = \left(\frac{V_i}{V_o}\right)^{1-k}
\]

\[
\frac{P_i}{P_o} = \left(\frac{V_o}{V_i}\right)^{k}
\]

\[
\frac{1}{P_o} = \left(\frac{1}{P_i}\right)^{k} = \left(\frac{P_i}{P_o}\right)^{k}
\]

\[
\text{writing differential form:} \\
\ln\left(\frac{P_i}{P_o}\right) = k \ln\left(\frac{P_i}{P_o}\right)
\]

\[
\frac{dP}{P} = k dP
\]

\[
\frac{dP}{p} = \frac{kP}{p} = k \frac{P}{P}
\]

where \( \frac{dP}{p} = C^2 \)

\[
C^2 = \frac{kP}{p} \quad \text{and} \quad \frac{p}{p} = \frac{p}{p} = \frac{p}{p}
\]

\[
C = \sqrt{\frac{kR}{T}}
\]

Mach number becomes:

\[
Ma = \frac{v}{C} = \frac{v}{\sqrt{\frac{dP}{dp}}} \rightarrow \text{all fluids}
\]

\[
Ma = \sqrt{\frac{P}{kRT}} \rightarrow \text{ideal gas}
\]
2. Fundemental Laws used in Turbomachinery

2.7. Compressible flow relations

→ Above combinations yield to many definitions used in turbomachinery for compressible flow. Some are listed below:

1. Stagnation temperature – pressure relation between two arbitrary points:

\[
\frac{P_{02}}{P_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}
\]

2. Capacity (non-dimensional flow rate):

\[
\frac{m\sqrt{C_pT_0}}{A_nP_0} = \frac{\gamma}{\sqrt{\gamma-1}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{2} \left(\frac{\gamma+1}{\gamma-1}\right)}
\]

3. Relative stagnation properties and Mach #:

\[
\frac{p_{0,rel}}{p}, \frac{T_{0,rel}}{T}, \frac{\rho_{0,rel}}{\rho}, \frac{m\sqrt{C_pT_{0,rel}}}{A p_{0,rel}} = f(M_{rel})
\]

HOMEWORK: Derive the non dimensional flow rate (Capacity) equation using equations (iii), (v) from the previous slide and the continuity equation
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

2.7. Compressible flow relations

Relation of static-relative-stagnation temperatures on a T-s diagram

Temperature – gas properties relation
2. Fundemental Laws used in Turbomachinery

2.8. Efficiency definitions used in Turbomachinery

1. Overall efficiency
\[ \eta_0 = \frac{\text{mechanical energy available at coupling of output shaft in unit time}}{\text{maximum energy difference possible for the fluid in unit time}} \]

2. Isentropic – hydraulic efficiency:
\[ \eta_t (\text{or } \eta_h) = \frac{\text{mechanical energy supplied to the rotor in unit time}}{\text{maximum energy difference possible for the fluid in unit time}} \]
\[ \eta_t (\text{or } \eta_h) = \frac{\text{actual work}}{\text{ideal (maximum) work}} = \frac{\Delta W_x}{\Delta W_{\text{max}}} \]

3. Mechanical efficiency:
\[ \eta_m = \eta_0 / \eta_t (\text{or } \eta_0 / \eta_h) \]

2.8.1 Steam and Gas Turbines

\[ \Delta W_x = \dot{W}_x / \dot{m} = h_{01} - h_{02} = (h_1 - h_2) + \frac{1}{2} (c_1^2 - c_2^2) \]

\[ \Delta W_{\text{max}} = \dot{W}_{\text{max}} / \dot{m} = h_{01} - h_{02s} = (h_1 - h_{2s}) + \frac{1}{2} (c_1^2 - c_{2s}^2) \]

1. The adiabatic total-to-total efficiency is:
\[ \eta_{tt} = \frac{\Delta W_x}{\Delta W_{\text{max}}} = \frac{(h_{01} - h_{02})}{(h_{01} - h_{02s})} \]

When inlet-exit velocity changes are small: :
\[ \eta_{tt} = \frac{(h_1 - h_2)}{(h_1 - h_{2s})} \]
Introduction to Turbomachinery

2. Fundemental Laws used in Turbomachinery

2.8.1 Steam and Gas Turbines

Enthalpy – entropy relation for turbines and compressors
Introduction to Turbomachinery

2. Fundamental Laws used in Turbomachinery

2. Total-to-static efficiency:

\[
\Delta W_{\text{max}} = \dot{W}_{\text{max}}/\dot{m} = h_{01} - h_{2s} = (h_1 - h_{2s}) + \frac{1}{2} c_1^2
\]

\[
\eta_{ts} = \Delta W_x/\Delta W_{\text{max}} = (h_{01} - h_{02})/(h_{01} - h_{2s})
\]

Note: This efficiency definition is used when the kinetic energy is not utilized and entirely wasted. Here, exit condition corresponds to ideal- static exit conditions are utilized \((h_{2s})\)

2.8.2. Hydraulic turbines

1. Turbine hydraulic efficiency

\[
\eta_h = \frac{\dot{W}_x}{\dot{W}_{\text{max}}} = \frac{\Delta W_x}{g[H_1 - H_2]}
\]

\[
\dot{W}_{\text{max}} = \dot{m} \left[ \frac{1}{\rho} (p_1 - p_2) + \frac{1}{2} (c_1^2 - c_2^2) + g(z_1 - z_2) \right] = \dot{m} g(H_1 - H_2)
\]

\[
gH = pl/\rho + \frac{1}{2} c^2 + gz \text{ and } \dot{m} = \rho Q
\]
2. Fundamental Laws used in Turbomachinery

2.8.3. Pumps and compressors

1. Isentropic (hydraulic for pumps) efficiency

\[ \eta_c \text{ (or } \eta_h) = \frac{\text{useful (hydraulic) energy input to fluid in unit time}}{\text{power input to rotor}} \]

2. Overall efficiency

\[ \eta_o = \frac{\text{useful (hydraulic) energy input to fluid in unit time}}{\text{power input to coupling of shaft}} \]

3. Total-to-total efficiency

\[ \eta_t = \frac{\text{ideal (minimum) work input}}{\text{actual work input}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \]

4. For incompressible flow:

\[ \Delta W_{\min} = \frac{\dot{W}_{\min}}{m} = \left( \frac{p_2 - p_1}{\rho} + \frac{1}{2} \left( c_2^2 - c_1^2 \right) + g(z_2 - z_1) \right) = g[H_2 - H_1] \]

\[ \eta_h = \frac{\dot{W}_{\min}}{\dot{W}_c} = \frac{g[H_2 - H_1]}{\Delta W_c} \]
2. Fundamental Laws used in Turbomachinery

2.8.4. Small Stage (Polytropic Efficiency) for an ideal gas For energy absorbing devices

\[ \eta_p = \frac{dh_{is}}{dh} = \frac{vdp}{C_p dT} \rightarrow \ \ C_p = \gamma R/(\gamma - 1) \rightarrow \ \ \frac{dT}{T} = \frac{(\gamma - 1) dp}{\gamma \eta_p p} \]

integrating \[ \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\eta_p \gamma} \]

For an ideal compression process \( \eta_p = 1 \), so

\[ \frac{T_{2s}}{T_1} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\gamma} \]

Therefore, the compressor efficiency is:

\[ \eta_c = \frac{(T_{2s} - T_1)}{(T_2 - T_1)} \]

\[ \eta_c = \left[ \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\gamma} - 1 \right] / \left[ \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\eta_p \gamma} - 1 \right] \]

NOTE: Polytropic efficiency is defined to show the differential pressure effect on the overall efficiency, resulting in an efficiency value higher than the isentropic efficiency.
2. Fundamental Laws used in Turbomachinery

2.8.5. Small Stage (Polytropic Efficiency) for an ideal gas For energy extracting devices

\[
\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n_p(\gamma-1)}{\gamma}},
\]

\[
\eta_s = \frac{1 - \left( \frac{p_2}{p_1} \right)^{\frac{n_p(\gamma-1)}{\gamma}}}{1 - \left( \frac{p_2}{p_1} \right)^{\frac{(\gamma-1)}{\gamma}}}
\]